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LETTER TO THE EDITOR

Renormalization group study of the sliding Luttinger liquids

Germán Sierra[1] **and Eugene H Kim**[2]

[1] Instituto de Física Teórica, CSIC/UAM, Madrid, Spain
[2] Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, Canada

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Abstract
We derive the RG flow equations of the sliding Luttinger liquid perturbed by charge-density-wave (CDW) and superconducting (SC) operators. Using them we study the phase diagram of an array of XXZ spin chains coupled by Ising terms. In the weak-coupling regime we find a new class of self-dual and antiself-dual unstable fixed points.

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1. Introduction

The Luttinger liquid (LL) is the key concept to describe interacting electrons in one dimension. The collective nature of the excitations, the spin-charge separation and the power-law behaviour of correlators with anomalous exponents are some of its distinctive features, in contrast to the Fermi liquid theory [1]. Considerable theoretical activity in the last few years has been devoted to the search of non-Fermi liquid theories in dimensions higher than one, specially in 2D, due to its possible connection with the high-T_c superconductors and another strongly correlated system.

A natural path to realize a non-Fermi liquid has been to couple arrays of 1D Luttinger liquids forming ladders and 2D planes. However, the general consensus has been, until recently, that 2D arrays of LL are unstable to the formation of crystal, superconducting or 2D Fermi liquid states [2]. An alternative to these 'no-go theorems' has been proposed lately using the concept of sliding Luttinger liquid (SLL), also called smectic non-Fermi liquid [3–5].

These works were partially motivated by the Anderson proposal of confinement of excitations in the Luttinger liquids [6] and have a classical analogue in the stacks of 2D XY models coupled by gradient interactions [7]. The SLL model may also be relevant to the stripe phases of the quantum Hall effect and the cuprates.

The sliding Luttinger liquid is the fixed point of a gaussian Hamiltonian which treats on equal footing the individual Luttinger Hamiltonians of the stripes and the density–density and current–current interstripe interactions [3, 4]. Using bosonization techniques one can regard the SLL as a set of decoupled LL characterized by a sound velocity $v(q_\perp)$ and a Luttinger coupling $K(q_\perp)$ which depend on the tranverse momentum $q_\perp$ across the stripes. The gaussian nature of the SLL-fixed point allows a simple derivation of the scaling dimensions of the single-particle (SP), charge-density (CDW) and superconducting (SC) order parameters, in terms of the SLL function $K(q_\perp)$ [3, 4]. These scaling dimensions have been used to study the stability of the SLL under various perturbations, finding rich phase diagrams where the SLL phase survives in the vicinity of CDW, SC and Fermi liquid phases [3–5]. The perturbative RG analysis performed in the latter references takes into account the running of the coupling constants of the perturbations to first order, while the Luttinger functions $K(q_\perp)$ and $v(q_\perp)$ stay constant. It is however well known that in certain situations, as in the presence of marginal perturbations, one has also to consider the renormalization of the Luttinger parameters in a RG in the style of Kosterlitz–Thouless (KT) [8]. This is, for example, the case of the Hubbard model at half filling where the Umklapp operator becomes marginally relevant for a repulsive Hubbard constant, leading to a charge gap in the low-energy spectrum.

The aim of this paper is to derive the one-loop RG equations of the SLL for the coupling constants and the SLL functions, and study some of their consequences in a model consisting of arrays of XXZ spin chains coupled by Ising terms. The latter model has been treated in the past with bosonization [12], mean field [13] and variational methods [14], which predict the existence of large regions in parameter space where the phase is either Ising-like or XY-like, corresponding respectively to the smectic crystal and the smectic metallic phases of [3, 4].

On more general grounds we also analyse the stability of the spinless SLL under relevant and marginal CDW and SC perturbations, finding new non-gaussian fixed points. Our results are related to those obtained by Boyanovsky and Holman [9] who have studied a class of extended sine-Gordon models based on simply-laced Lie groups (see also [10]).

2. The sliding Luttinger model

Let us consider an array of N spinless Luttinger stripes with phase fields for the density fluctuations ϕ_a $(a = 1, \ldots, N)$ and Euclidean Lagrangian

$$\mathcal{L}_0 = \sum_{a=1}^{N} \frac{K_0}{2} \left[\frac{1}{v_0} (\partial_t \phi_a)^2 + v_0 (\partial_x \phi_a)^2 \right] \tag{1}$$

where K_0 is the inverse of the standard Luttinger parameter ($K_0 > 1$ for repulsion) and v_0 is the sound velocity which we scale to 1. The charge density fluctuations j_0^a and the charge currents j_1^a are given by the bosonization equation $j_\mu^a = \frac{1}{\pi} \epsilon_{\mu\nu} \partial^\nu \phi_a$. Hence the density–density and current–current interactions among the stripes are also quadratic in the derivatives of the bosonic fields and, together with (1), define the sliding Luttinger Lagrangian [3, 4]

$$\mathcal{L}_{\mathrm{SLL}} = \frac{1}{2} \sum_{a,b=1}^{N} \left[\partial_t \phi_a K_{a,b}^{J} \partial_t \phi_b + \partial_x \phi_a K_{a,b}^{\rho} \partial_x \phi_b \right] \tag{2}$$

where $K_{a,b}^{J,\rho} = \delta_{a,b} K_0 + \bar{K}_{a,b}^{J,\rho}$ are $N \times N$ matrices whose off-diagonal elements are given by the interstripe current–current and density–density interactions. The SLL model can alternatively be formulated in the dual variables θ_a, which are the phase fields of the superconducting fluctuations. The SLL Lagrangian (2) becomes [3–5]

$$\mathcal{L}_{\mathrm{SLL}} = \frac{1}{2} \sum_{a,b=1}^{N} \left[\partial_t \theta_a \left(K_\rho^{-1} \right)_{a,b} \partial_t \theta_b + \partial_x \theta_a \left(K_J^{-1} \right)_{a,b} \partial_x \theta_b \right] \tag{3}$$

where $K_{J,\rho}^{-1}$ are the inverse matrices of $K^{J,\rho}$. Both equations (2) and (3) exhibit the smectic or sliding symmetries $\phi_a \to \phi_a + \alpha_a$ and $\theta_a \to \theta_a + \beta_a$, where α_a and β_a are constants, which prevents the lock-in of the charge-density-wave and superconducting order parameters of the individual stripes [3, 4]. Assuming periodic boundary conditions across the stripes and translational invariance along them, one can perform the Fourier transform: $\phi_a = \frac{1}{\sqrt{N}}\sum_{q_\perp} \mathrm{e}^{\mathrm{i}q_\perp a}\phi_{q_\perp}$, $K^{J,\rho}(q_\perp) = \sum_a \mathrm{e}^{\mathrm{i}q_\perp a} K^{J,\rho}_{1,1+a}$ in order to bring the SLL Lagrangian (2) into the form

$$\mathcal{L}_{\mathrm{SLL}} = \frac{1}{2}\sum_{q_\perp} K(q_\perp)\left[\frac{1}{v(q_\perp)}|\partial_t\phi_{q_\perp}|^2 + v(q_\perp)|\partial_x\phi_{q_\perp}|^2\right] \tag{4}$$

where $K(q_\perp) = \sqrt{K^J(q_\perp)K^\rho(q_\perp)}$ is the (inverse) Luttinger parameter and $v(q_\perp) = \sqrt{K^\rho(q_\perp)/K^J(q_\perp)}$ is the sound velocity of the $q_\perp$ mode. In the dual variables $\theta_{q_\perp}$ the Lagrangian has the same form as equation (4) with the replacement $K(q_\perp) \to 1/K(q_\perp)$. The scaling dimension of a generic vertex operator $V^\phi_\beta = \exp\left(\mathrm{i}\sum_a \beta_a\phi_a\right)$ is given by [3, 4]

$$\Delta^\phi_\beta = \int_{q_\perp} \frac{1}{4\pi K(q_\perp)}\left(\sum_a \beta_a^2 + 2\sum_{a<b}\beta_a\beta_b\cos(q_\perp(a-b))\right) \tag{5}$$

where $\int_{q_\perp} = \int_{-\pi}^{\pi}\frac{\mathrm{d}q_\perp}{2\pi}$. Similary, a vertex operator in the dual variables, i.e. $V^\theta_\beta = \exp\left(\mathrm{i}\sum_a \beta_a\theta_a\right)$, has a scaling dimension Δ^θ_β given by the formula (5) with $K(q_\perp)$ replaced by its inverse. The interaction Lagrangian is given by the pair hopping (SC) and particle–hole (CDW) operators [3, 4],

$$\mathcal{L}_{\mathrm{int}} = \int \frac{\mathrm{d}^2x}{(2\pi a_0)^2}\sum_{a,n>0}[g_{CD,n}\cos\beta(\phi_a - \phi_{a+n}) + g_{SC,n}\cos\beta(\theta_a - \theta_{a+n})] \tag{6}$$

where a_0 is the lattice spacing and $\beta^2 = 2\pi M$ with M an integer. For the charge modes of spin-gapped systems [3] one has $M = 1$, while for spinless fermions [4] one has $M = 2$.

In the latter references the stability of the perturbed SLL was studied in terms of the relevance or irrelevance of the CDW and SC operators (6), given by their scaling dimensions. There are, however, cases where one has to consider the renormalization of the functions $K(q_\perp)$ and $v(q_\perp)$, for example when the CDW and SC operators become marginal. This problem is addressed in the next section.

3. RG equations for the SLL model

Equations (2) and (6) define a multicomponent sine-Gordon model which can be renormalized using operator product expansion (OPE) techniques [11]. This renormalization procedure is simplified if we assume that the sound velocity $v(q_\perp)$ is the same for all the modes and set it equal to 1. To present our results we shall expand the Luttinger function as $K(q_\perp) = 1 + k_0 + \sum_{n>0} k_n\cos(q_\perp n)$ and assume that the parameters k_n are small. The one-loop RG equations for the model with Lagrangian $\mathcal{L}_{\mathrm{SLL}} + \mathcal{L}_{\mathrm{int}}$ are given by

$$\begin{aligned}
\frac{\mathrm{d}k_0}{\mathrm{d}s} &= \frac{M}{(4\pi)^2}\sum_{n>0}\left(g^2_{CD,n} - g^2_{SC,n}\right)\\
\frac{\mathrm{d}k_n}{\mathrm{d}s} &= -\frac{M}{(4\pi)^2}\left(g^2_{CD,n} - g^2_{SC,n}\right) \qquad (n>0)\\
\frac{\mathrm{d}g_{CD,n}}{\mathrm{d}s} &= \left(2 - M + M\left(k_0 - \frac{k_n}{2}\right)\right)g_{CD,n} - \frac{1}{4\pi}\frac{\partial\mathcal{N}(g_{CD})}{\partial g_{CD,n}}\\
\frac{\mathrm{d}g_{SC,n}}{\mathrm{d}s} &= \left(2 - M - M\left(k_0 - \frac{k_n}{2}\right)\right)g_{SC,n} - \frac{1}{4\pi}\frac{\partial\mathcal{N}(g_{SC})}{\partial g_{SC,n}}
\end{aligned} \tag{7}$$

where $\mathcal{N}(g)$ is the cubic polynomial

$$\mathcal{N}(g) = \sum_{n,m>0} g_n g_m g_{n+m} \tag{8}$$

which encodes the OPE of the CDW and SC operators appearing in (6). From (7) the RG conservation of $K(q_\perp = 0) = 1 + \sum_{n\geqslant 0} k_n$ follows.

The RG equation (7) can be easily generalized to include other types of operators such as the Umklapp ones, i.e. $\cos\beta(\phi_a + \phi_b)$ and their duals, i.e. $\cos\beta(\theta_a + \theta_b)$. This has been partially done by Boyanovsky and Holman (BH) for a family of extended sine-Gordon models based on simply-laced Lie algebras $\mathcal{G}$ [9]. BH have considered all the vertex operators of the form $\exp(\beta\alpha_a \cdot \phi)$ and their duals $\exp(\beta\alpha_a \cdot \theta)$, where α_a are all the roots of $\mathcal{G}$. However, the kinetic term in [9] does not include inter-chain forward scattering terms, i.e. $k_n = 0$ $(\forall n > 0)$, so that only k_0 is allowed. This imposes severe restrictions on the coupling constants g_{α_a} and their duals $\tilde{g}_{\alpha_a}$ in order to maintain renormalizability, so that one is left only with a single coupling constant g and its dual $\tilde{g}$, apart from discrete choices of signs. In this sense our work generalizes the approach of BH placing it in the framework of SLL models. Conversely, the use of group theoretical methods may help us in understanding the renormalizability properties of the SLL models.

In this respect, it may be worth mentioning that the model defined by equations (2) and (6) is connected to the Lie group $SO(2N)$, where N is the number of legs. This group has $N(2N-1)$ generators which have the following meaning. The Cartan subalgebra, which has dimension N, gives the bosons, $\{\phi_a\}_{a=1}^N$, of the model. The $N(N-1)$ roots of the form $\eta_{i,j} = (\mathbf{e}_i - \mathbf{e}_j)/\sqrt{2}$ give the CDW or the SC operators in (6) ($\mathbf{e}_i = (0, \ldots, 1, \ldots, 0)$ is a N-component unit vector). Finally, the $N(N-1)$ roots of the form $\pm\lambda_{i,j} = \pm(\mathbf{e}_i + \mathbf{e}_j)/\sqrt{2}$ correspond to Umklapp operators. Dropping the latter type of operators reduces the symmetry from $SO(2N)$ to its maximal compact subgroup $SU(N) \otimes U(1)$.

4. Arrays of coupled XXZ spin chains

As a first application of the previous results, let us consider a system of coupled XXZ spin chain Hamiltonians via Ising and spin-pair-flipping terms [12, 14],

$$\begin{aligned} H = \sum_{i=1}^{L}\sum_{a=1}^{N}\bigg[&-\frac{J}{2}\big(S^+_{i,a}S^-_{i+1,a} + S^-_{i,a}S^+_{i+1,a}\big) + J_z S^z_{i,a}S^z_{i+1,a} + J'_z S^z_{i,a}S^z_{i,a+1} \\ &+ J'_{XY}\big(S^+_{i,a}S^+_{i+1,a}S^-_{i,a+1}S^-_{i+1,a+1} + \mathrm{hc}\big)\bigg]. \end{aligned} \tag{9}$$

This model can be Jordan–Wigner transformed into a spinless fermion Hamiltonian, i.e. $M = 2$, which upon bosonization becomes at half-filling [12],

$$\begin{aligned} H = \sum_{a=1}^{N}\int \mathrm{d}x \bigg[&\frac{u}{2K_0}\pi_a^2 + \frac{uK_0}{2}(\partial_x\phi_a)^2 + \frac{2J_z a_0}{(2\pi a_0)^2}\cos\sqrt{16\pi}\phi_a \\ &+ \frac{J'_z a_0}{\pi}\partial_x\phi_a\partial_x\phi_{a+1} + \frac{8J'_{XY}a_0}{(2\pi a_0)^2}\cos\sqrt{4\pi}(\theta_a - \theta_{a+1}) \\ &+ \frac{2J'_z a_0}{(2\pi a_0)^2}(\cos\sqrt{4\pi}(\phi_a + \phi_{a+1}) - \cos\sqrt{4\pi}(\phi_a - \phi_{a+1}))\bigg] \end{aligned} \tag{10}$$

where $u = Ja_0(1 + 2J_z/(\pi J)$ and $K_0 = 1 + 2J_z/(\pi J)$ for $|J_z| \ll J$. The gaussian terms in (10) yield a SLL model (2) with

$$K^J_{a,b} = \delta_{a,b} K_0 \qquad K^{\rho}_{a,b} = \delta_{a,b} K_0 + \frac{J'_z}{\pi}\delta_{|a-b|,1} \tag{11}$$

where the time variable and the exchange couplings are measured in units of u and J, respectively. In the weak-coupling regime $|J_z| \ll J$, the SLL function $K(q_\perp)$ is close to 1. Consequently, the intra-chain Umklapp couplings $\cos\sqrt{16\pi}\phi_a$ have dimension $\Delta \sim 4$ and hence can be neglected, as is the case of decoupled single chains. On the other hand, the CDW, SC and Umklapp inter-chain couplings are marginal and one has to consider their running together with that of the SLL functions.

To simplify matters we shall neglect in what follows the Umklapp term, which is absent away from half-filling, and the SC term which have not been considered in [12, 14]. The effective model which is left is given by the following set of non-vanishing couplings,

$$M = 2 \qquad k_0 = \frac{2J_z}{\pi} \qquad k_1 = \frac{J'_z}{\pi} \qquad g_{CD,1} = -2J'_z \tag{12}$$

and the RG equations (7) become

$$\begin{aligned} \frac{\mathrm{d}k_0}{\mathrm{d}s} &= \frac{1}{8\pi^2}\sum_{n>0} g^2_{CD,n} \\ \frac{\mathrm{d}k_n}{\mathrm{d}s} &= -\frac{1}{8\pi^2} g^2_{CD,n} \qquad (n > 0) \\ \frac{\mathrm{d}g_{CD,n}}{\mathrm{d}s} &= (2k_0 - k_n) g_{CD,n} - \frac{1}{4\pi}\frac{\partial \mathcal{N}(g_{CD})}{\partial g_{CD,n}}. \end{aligned} \tag{13}$$

Let us first consider the simplest case where only k_0 and $g_{CD,1}$ are non-vanishing. A truncation of equations (13), written in the variables $x = -2k_0$ and $y = g_{CD,1}/(2\pi)$, gives

$$\frac{\mathrm{d}x}{\mathrm{d}s} = -y^2 \qquad \frac{\mathrm{d}y}{\mathrm{d}s} = -xy. \tag{14}$$

These are the well-known Kosterlitz–Thouless RG equations of the XY model [8, 11], whose RG trajectories are hyperbolas which end up at infinity provided $|y| > x$. Using equations (12), the latter inequality becomes $|J'_z| > -4J_z$. This condition coincides with the one derived in [12] for the existence of an Ising phase in the XXZ + Ising model (see figure 1). In the weak-coupling regime of the XY model, the parameter y flows to zero and x flows to the line $x > 0$ of stable fixed points. In [12], the inter-chain forward scattering couplings were not taken into account. To do so, we have solved numerically equations (13) with the initial conditions given by equation (12). The phase diagram is shown in figure 1. We see that the boundary separating the Ising and XY regions, which in the previous computation was given by $J'_z = \pm 4|J_z|$, $J_z < 0$, is still linear but the slopes have been modified to $J'_z = 5.828|J_z|$ and $J'_z = -1.750|J_z|$ with $J_z < 0$. This is an effect of the inclusion of the forward scattering terms k_n and the additional coupling constants $g_{CD,n}$.

In the XY region all the couplings g_n flow to zero, while the couplings k_n flow to a fixed value. Thus in this region the low energy of the XXZ + Ising model is described by a SLL model. As can be seen from figure 1 a necessary condition to achieve a XY phase is to have a ferromagnetic intra-chain coupling $J_z < 0$, which agrees with the results of [4] concerning the proximity of the stable SLL to the isotropic ferromagnetic point where the boson stiffness K_0 vanishes.

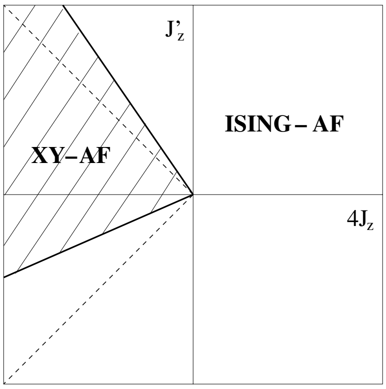

Figure 1. Phase diagram of the XXZ + Ising model. The shadow region denotes the XY-AF phase and it is given by $-1.750|J_z| < J_z' < 5.828|J_z|$ and $J_z < 0$. The region inside the dotted lines, $-4|J_z| < J_z' < 4|J_z|$ and $J_z < 0$, is obtained by neglecting the inter-chain forward scattering terms k_n ($n \geqslant 1$) and the higher-order couplings $g_{CD,n}$ ($n > 1$).

5. Fixed points of the SLL model

Another application of equations (7) is the search for non-trivial fixed points. This may happen whenever the couplings $g_{CD,n}$ and g_{SC_n} are either relevant, i.e. $M < 2$, or marginal, i.e. $M = 2$. We shall consider these two cases separately.

5.1. Case $M < 2$

Equations (7) have the following interesting property. Let us suppose that $g_{CD,n} = g_{SC,n}$ and that $k_0 = k_n/2$ for all values of $n > 0$ at a given scale, say $s = 0$. Then the self-duality conditions $g_{CD,n} = g_{SC,n}$ are preserved by the RG flow, and k_n ($\forall n \geqslant 0$) stay constant and drop from the RG equations of $g_{CD,n} = g_{SC,n}$. The RG equations for the couplings $g_n \equiv g_{CD,n}/(2\pi) = g_{SC,n}/(2\pi)$ are given by

$$\frac{\mathrm{d}g_n}{\mathrm{d}s} = g_n\delta - \frac{1}{2}\frac{\partial\mathcal{N}(g)}{\partial g_n} \tag{15}$$

where $\delta = 2 - M$. A similar observation was made by BH for the extended sine-Gordon models mentioned above [9]. Equation (15) can be more conveniently written in terms of the matrix $g_{a,b} = g_{|a-b|}$, $g_{a,a} = 0$ as

$$\frac{\mathrm{d}g_{a,b}}{\mathrm{d}s} = g_{a,b}\delta - \frac{1}{2}\sum_{c=1}^{N} g_{a,c}g_{c,b}. \tag{16}$$

We can further assume that all the couplings are actually the same, i.e. $g_n = g$, so that (16) yields

$$\frac{\mathrm{d}g}{\mathrm{d}s} = g\delta - \frac{N-2}{2}g^2. \tag{17}$$

This equation has a stable fixed point $g = g_*$ given by

$$g_* = \frac{2\delta}{N-2} \tag{18}$$

which is analogue to the ones found by BH, under the asumption that all the couplings are the same [9].

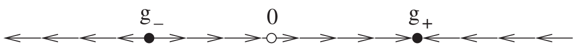

Figure 2. RG flow of the coupling constants $g(q_\perp)$.

We can look for the general fixed points of equation (16) without imposing the condition that all g_n are equal. To do so we introduce the Fourier transform $g(q_\perp) = \sum_a \mathrm{e}^{\mathrm{i}q_\perp a} g_{1,1+a}$ and write equation (16) as

$$\begin{aligned} \frac{\mathrm{d}g(q_\perp)}{\mathrm{d}s} &= g(q_\perp)\delta - \frac{1}{2}g(q_\perp)^2 + \frac{1}{2}A \\ A &= \frac{1}{N}\sum_{q_\perp} g(q_\perp)^2. \end{aligned} \tag{19}$$

Note that $\sum_{q_\perp} g(q_\perp) = 0$ because $g_{a,a} = 0$. Equation (19) seems to imply that the RG flows of the modes of $g(q_\perp)$ are independent. However, this is not true due to the term A which couples all the modes. For each value of $q_\perp$ the fixed points of (19) are given by one of the roots $g_\pm$ of the quadratic equation $g(q_\perp)\delta - \frac{1}{2}g(q_\perp)^2 + A = 0$, namely,

$$g_\pm = \delta \pm \sqrt{\delta^2 + A}. \tag{20}$$

Let us denote by n_+ and n_- the number of $q'_\perp$ for which $g(q_\perp)$ is either g_+ or g_-. One has of course $n_+ + n_- = N$. Introducing (20) into the definition of A one finds the expression of $g_\pm$ in terms of $n_\pm$, i.e.

$$g_+ = \frac{2\delta n_-}{n_- - n_+} \qquad g_- = -\frac{2\delta n_+}{n_- - n_+} \tag{21}$$

and in turn the value of A

$$\sqrt{\delta^2 + A} = \frac{N\delta}{n_- - n_+} \tag{22}$$

which yields the condition $n_+ < n_-$. The couplings g_+ are stable, while the couplings g_- are unstable (see figure 2)

The fixed point $g_n = g_*$ obtained previously (see equation (18)) gives $g(q_\perp) = g_*(N\delta_{q_\perp,0} - 1)$, and hence it corresponds to the choice $(n_+, n_-) = (1, N-1)$. Consequently, there is one stable direction and $N-1$ unstable ones in the full g_n space of couplings.

For a given pair (n_+, n_-), satisfying $n_+ + n_- = N$ and $n_+ < n_-$, there are $\binom{N}{n_+}$ fixed points, corresponding to all possible choices $g(q_\perp) = g_+$ or g_-.

Another example is given by the fixed point $(n_+, n_-) = \left(\frac{N-1}{2}, \frac{N+1}{2}\right)$, with N odd and $\{g(q_\perp)\} = (g_+, \overset{n_+}{\ldots}, g_+, g_-, \overset{n_-}{\ldots}, g_-)$. The corresponding values of $g_{*,n}$ are given by

$$g_{*,a} = 2\delta \frac{\sin\left(\frac{\pi a}{2}\frac{N-1}{N}\right)}{\sin\left(\frac{\pi a}{N}\right)} \qquad a = 1, \ldots, N \tag{23}$$

which for $N \to \infty$ implies $g_{*,2a} = 0$ and $g_{2a+1} \sim (-1)^a/(2a+1)$.

5.2. $M = 2$

For $M = 2$ the self-dual conditions $g_{CD,n} = g_{SC,n} \equiv 2\pi g_n$ are also preserved by the RG flow, provided $k_0 = k_n/2$. The RG equations for g_n are given by equation (15) with $\delta = 0$. The fixed points can be found by the same procedure employed earlier. They correspond to the case $n_+ = n_- = N/2$ (N even) and $g_+ = -g_-$.

Equations (7) have other fixed points at $M = 2$, which do not exist for $M < 2$. They satisfy the antiself-dual constraints $g_{CD,n} = -g_{SC,n} \equiv 2\pi g_{*,n}$ and

$$0 = (2k_0 - k_n)g_{*,n} - \frac{1}{2}\frac{\partial \mathcal{N}(g_*)}{\partial g_{*,n}} \tag{24}$$

where k_0 and k_n are arbitrary. However, the antiself-dual constraint $g_{CD,n} = -g_{SC,n}$ is not preserved by the RG flow (7), even in the case where $k_0 = k_n/2$. The antiself-dual fixed points (24) are also unstable.

6. Conclusions

In this paper we have presented the one-loop RG equations of the sliding Luttinger liquid (SLL) model perturbed by charge-density-wave (CDW) and superconducting (SC) operators.

These equations have been applied to determine the phase diagram of an array of XXZ spin chains coupled by Ising terms, finding a XY phase and an Ising phase. The XY phase is described by a SLL model with renormalized values of the SLL parameters.

We have also found new non-gaussian fixed points of the SLL-RG equations. In the case where the CDW and SC are relevant these fixed points are self-dual, i.e. $g_{CD,n} = g_{SC,n}$, while for marginal operators there are self-dual and antiself-dual fixed points, $g_{CD,n} = -g_{SC,n}$. The precise nature of these fixed points is an interesting problem to be clarified in the future using more powerful techniques as perturbative conformal field theory and the c-theorem [10]. The generalization of our results to SLL with charge and spin degrees of freedom is rather straightforward. Here too we expect the appearance of novel non-gaussian fixed points.

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